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LETTER TO THE EDITOR

Fractal dimensionality and the number of visited sites of the ant in the labyrinth

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Abstract. Monte Carlo studies in three dimensions confirm the hypothesis of Toulouse and Rammal that the number of distinct sites visited by a random walk on a random network at its percolation threshold varies as $(\text{time})^{2/3}$.

The largest cluster at the percolation threshold is recognised to be a fractal because of its self-similar nature (Mandelbrot 1983). Numerous studies of random walks on these structures have been made recently; see Kehr and Binder (1983) and Mitescu and Roussenq (1983) for reviews. The critical exponent of the random walk is related to the critical exponent μ (often also denoted as t) of the conductivity of random resistor networks. Alexander and Orbach (1982) related μ to other critical exponents of percolation theory by asserting for the average end-to-end distance r after very long times t:

$$r \propto t^{2/3D}. \tag{1}$$

Here $D = d/(1+1/\delta)$ is the fractal dimension of the largest cluster at the d-dimensional percolation threshold. Rammal and Toulouse (1983) made an additional assumption leading to

$$S(t) \propto t^{2/3} \tag{2}$$

where S is the number of distinct sites visited by the random walk in t steps. Monte Carlo simulations confirmed this exponent $\frac{2}{3}$ (which can be called half the spectral dimensionality) reasonably well in two dimensions (Angles d'Auriac *et al* 1983) and on the Bethe lattice (Angles d'Auriac and Rammal 1983). For other diffusion problems, some deviations from scaling theory were reported by Mitescu and Roussenq (1983) in three dimensions; these deviations seem to be due to the short times t employed there, since Pandey and Stauffer (1983) found agreement between Monte Carlo simulations and scaling theory for $t \sim 10^6$. The confirmations of scaling theory for $S(t)$ are based on shorter time $t \sim 10^4$ (Angles d'Auriac *et al* 1983, Angles d'Auriac and Rammal 1983). Thus the present work checks on the simple cubic lattice, whether Monte Carlo data for long times agree with the scaling theory for $S(t)$, or whether earlier agreement was based on a cancellation of theoretical errors and systematic simulation errors.

We start our diffusion process for the 'ant in the labyrinth' at an arbitrary occupied site of the lattice with occupation probability $p = 0.3117 = p_c$. Thus we have to average over both finite clusters and the 'incipient' infinite network. This averaging (Stauffer

1979) gives a slightly modified exponent (Angles d'Auriac and Rammal 1983):

$$S(t) \propto t^{(2/3)(1-1/\delta)}. \tag{3}$$

Our lattice size was 150^3 and smaller; we stored in single bits whether a site is occupied or empty, and whether it was visited before or not. A CDC Cyber 76 computer needed about 3.6 μs for each step on average. (For small lattices single-bit handling could be avoided, reducing the time to about 2 μs. We are trying to simulate the problem also on a Cyber 205 vector computer where 0.4 μs can be reached if only r, and not S, is measured.)

Figure 1 shows the Monte Carlo results. In spite of the long times used and the variation of lattice size we found no systematic change of our critical exponent for $S(t)$ with time or system size, in contrast to the case of end-to-end distances (Pandey and Stauffer 1983). We find from this preliminary analysis

$$S(t) \propto t^{0.54\pm0.02} \tag{4}$$

in good agreement with the theoretical exponent 0.54 ± 0.01 from equation (3). Thus the Rammal–Toulouse theory seems confirmed.

This result is also a rather direct confirmation of the Alexander–Orbach rule of equation (1). Whereas Rammal and Toulouse (1983) needed an additional assumption to relate equations (1) and (2), we now relate them more directly by scaling arguments. (R Rammal informed us that he has a different derivation of the links between equations (1) and (2).) Dynamical scaling (Gefen *et al* 1983) suggests near p_c for large walks on large clusters containing s sites each:

$$S(t) = t^x f(ts^y, (p-p_c)s^\sigma) \tag{5}$$

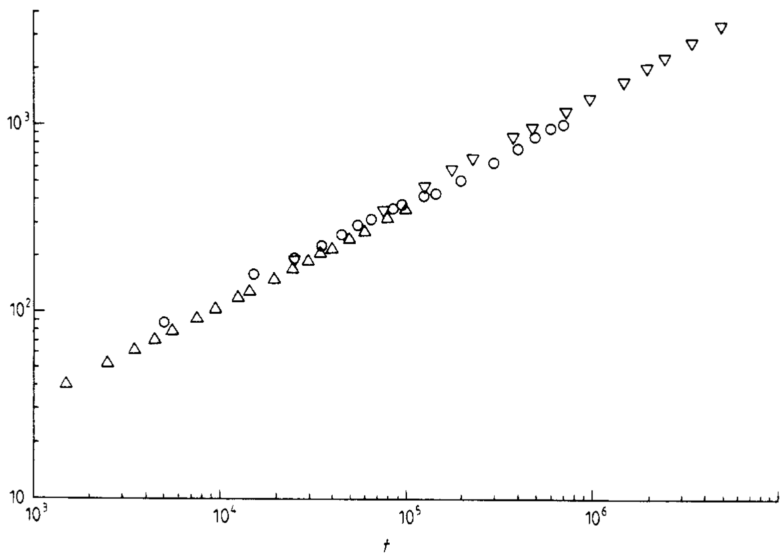

Figure 1. Number, S, of distinct visited sites plotted against time t (number of step attempts in a walk). The different symbols denote data taken from different samples. $\triangle$, sample size $L^3 = 150\times150\times150$, number of different random walks (starting points for the ants) $N = 100$ each on NRUN $= 50$ independent lattice realisations at the percolation threshold $p_c = 0.3117$ for times up to 10^5; $\bigcirc$, $L = 90$, $N = 5$, NRUN $= 10$, $t \leq 10^6$; ∇, $L = 150$, $N = 10$, NRUN $= 25$, $t \leq \frac{1}{2}\,10^7$.

where $\sigma = 1/\beta\delta$, x is the exponent we want to determine, and y can be derived to be $y = -z/D = -[2+(\mu-\beta)/\nu]/D$ (Gefen *et al* 1983). Times scale as $(\text{length})^z$ and lengths scale as $s^{1/D}$. At the critical point $p = p_c$ we may rewrite equation (5) as

$$S(t) = t^x g(t/s^{z/D}). \tag{6}$$

For $t \to \infty$ we know that each site of the finite cluster will be visited:

$$S(t=\infty) = s. \tag{7}$$

Equations (6) and (7) together require the scaling function g to vanish for large arguments as $(t/s^{z/D})^{-x}$, with $x = D/z$. Equation (1) means $z = 3D/2$, and thus we rederived $x = \frac{2}{3}$. In this sense, our Monte Carlo data are a numerical confirmation of the Alexander–Orbach rule for μ and give a surprisingly accurate $\mu/\nu = 2.2 \pm 0.2$ with little computational effort, in agreement with other recent work (Mitescu and Greene 1983, Derrida *et al* 1983, Pandey and Stauffer 1983, see also Sahimi *et al* 1983).

In summary, our results are in full accord with our present understanding of kinetic percolation.

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